Target Localization with a Single Sensor via Multipath Exploitation

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Abstract—In urban sensing and through-the-wall radar, the existence of targets in proximity to walls or buildings results in multipath returns. The multipath from the walls is exploited to achieve target localization with a single sensor. A time-of-arrival (ToA) wall association algorithm is derived to relate multipath returns to their respective walls and targets, followed by a nonlinear least squares optimization. Simulations and experimental data are used to validate the proposed algorithms.

Keywords—Through-the-wall radar, multipath exploitation, target localization, nonlinear least squares, urban canyon, urban sensing.

I. INTRODUCTION

The existence of targets in close proximity to walls, floors and ceilings, as in the case of through-the-wall (TTW) sensing, introduces multipath returns in the received radar signal. These returns could be confused for false targets, causing high system false alarms. However, multipath can be exploited for target localization without the need to employ a physical or synthesized aperture. Instead, a single antenna radar system may be used. In essence, multipath returns may be utilized to provide additional virtual sensors permitting noncoherent target localization. The single sensor approach to urban target localization is a viable, cost effective solution for target localization in TTW radar as well as in urban canyon situations [1-20].

The conventional approach for TTW localization is via a synthetic aperture radar (SAR) based system comprising multiple sensors [5-12]. In [14], multipath was exploited, in the context of time reversal, to improve the beamformed image for a TTW scenario, but not for enlarged aperture. Noncoherent
localization using multiple sensors for a single target in a TTW environment was analyzed in [12, 13], and references therein, but multipath was not specifically analyzed or exploited for target localization. Multipath exploitation techniques have been reported in the literature [15-19]. In [15], a range Doppler map for a single target direct line-of-sight (LOS) path and its multipath in an urban canyon is provided; the authors used both the single bounce as well as the double bounce multipath and a model for the multipath was assumed. In [16], a statistical detection technique was advocated for a single moving target in an urban canyon exploiting the specular multipath, and an orthogonal frequency division multiplexing (OFDM) radar model was proposed for incorporating the multipath Doppler shifts. The authors in [17] evaluated the potential for utilizing specular multipath reflections to improve detection for an airborne system. In [18], it was demonstrated experimentally that indoor targets, such as rotating fans and a walking person with a metal reflector, are detected via their respective multipath only. In [19], target tracking via multipath exploitation and waveform selection was developed assuming a 3D model in an urban canyon. A state space formulation was derived and a particle filter was used to track the target. The model assumed either constant Doppler or turning motions for a single target.

Our technique is quite different from the methods discussed above. Localization was neither the objective nor was it discussed in [15-18], whereas in [19], the authors fuse both the Doppler and the range information of a single target in a model to perform tracking. Our method, on the other hand, does not include any Doppler information, nor is it model specific. However, in order to achieve single-sensor based stationary or moving urban target localization, our approach assumes knowledge of the layout of surrounding walls and reflecting surfaces. This knowledge may be available either through city and building blueprints or from prior surveillance operations [15-17, 19]. While surrounding walls in TTW sensing consist of side, front, and back walls, only two adjacent side walls are typically considered in urban canyon situations. We focus on single-sensor localization of stationary targets, which is performed after background subtraction so as to remove clutter and target-independent multipath returns. The proposed approach is also applicable to targets which change position with time. In this case, however, clutter can be suppressed using moving target indication (MTI) filters, such as delay line cancellers.
In this paper, we cast target-wall multipaths as emanating from virtual sensors. We initially demonstrate the principle of target localization for a single target scene by utilizing the specular reflections from a single wall. It is then shown that, in the presence of additional walls, correct associations of the multipath returns with their respective walls must be performed. To this end, we derive an algorithm to trace each multipath’s time-of-arrival (ToA) to its particular wall, and validate this association through simulation. For multiple targets, localization is achieved by sequentially grouping the ToAs for each target. The wall association routine is then applied to each group separately.

We primarily discuss target localization for the case of unobstructed LOS, i.e., in free-space. Since a front wall in TTW scenario encloses the target inside a rectangular room and separates it from the sensor, we also derive a nonlinear least squares (NLS) approach, which compensates for the wall propagation effects for accurate TTW target localization.

The analysis presented here considers two dimensional scenes with point targets, since this is sufficient to describe the technique. However, extensions to the third dimension, accounting for multipath reflections from the floor and ceiling, and to incorporate extended targets are a straightforward matter. The proposed techniques can be readily extended following the methodology in [9, 20].

The remainder of the paper is organized as follows. In Section II, the principle of localization via multipath is examined analytically, and a model incorporating the surrounding walls, namely two side walls and a back wall, is derived. We present an algorithm which allows the respective multipath ToAs to be associated correctly to their respective walls. In Section III, multiple targets are considered and a ToA clustering technique is derived through which target localizations are achieved. In Section IV, the devised techniques are tailored to address TTW target localization. Simulation and experimental results are provided in Section V, and conclusions are presented in Section VI.
II.  MOTIVATION & MODEL

A.  Motivation

Consider Fig. 1 which shows the sensor-target geometry. There is a single sensor (monostatic radar) at point $R$ and a target at point $A$, which is adjacent to a smooth reflecting wall (wall-1). The position vectors of the radar and the target are, respectively given by $\mathbf{R} = [-D_x, 0]^T$, and $\mathbf{x}_t = [-x_t, y_t]^T$, in the Cartesian coordinate system. We assume that the radar waveform is sufficiently wideband so that the multipath and direct path returns are resolved.

The direct path return corresponds to the signal that propagates to and from the target along the direct path, $RA$. In addition to the direct path, there is an indirect path, $RBA$, by which the signal may reach the target. This indirect path involves a reflection on the wall at point $B$. There are two types of multipath returns, which arise because of the different permutations of the round trip the signal may take to the target when path $RBA$ is considered. The first type corresponds to the first order multipath, in which the signal round trip consists of a leg along path $RA$, and a leg along path $RBA$. The order in which the signal propagates along the two paths is not important. Traveling out along $RA$ and returning along $RBA$ has the same ToA as traveling out along $RBA$ and returning along $RA$; both permutations, therefore, lead to a single return. The second type is the second order multipath, which travels to and from the target along path $RBA$. Depending on the context, both the first and second order multipath returns are referred to generally as multipath in the rest of the paper.

The smoothness assumption of wall-1 implies specular reflections at point $B$. As a result, we observe from Fig. 1 that reflecting the indirect path about wall-1 yields an alternative radar-target geometry. The point $R_1$ is defined as the virtual radar and $R_1B = RB$. Additionally, we obtain the virtual target denoted as $A_1$ and $A_1B = AB$. It is readily seen from the geometry that the virtual target and the virtual radar position vectors are given by $[x_t, y_t]^T$ and $[D_x, 0]^T$, respectively. Considering Fig. 2, we observe that the first order multipath has the same time delay as a bistatic configuration, comprising the radar and the virtual radar. That is, in terms of the range, we have
\[ RA + AB + BR = RA + RA_i = RA + R_i A \]  

where \( RA + R_i A \) represents the bistatic configuration, i.e., the signal propagates from the radar to the target, and is received by the virtual radar. The constant range contour corresponding to the first order multipath, represented by the red dashed line in the figure, is thus an ellipse, which has foci at the radar and the virtual radar and passes through the target and the virtual target locations. On the other hand, the round-trip along the direct path and the second order multipath, being monostatic measurements, have circular constant range contours (the dashed black and blue lines), centered, respectively, at the radar and the virtual radar, with both passing through the target location. Both the circles and the ellipse, therefore, intersect at the true target location \( x_t \). Mathematically, we seek a common intersection point of,

\[
\frac{x^2}{a_t^2} + \frac{y^2}{a_t^2 - c_t^2} = 1, \quad a_t = \frac{\sqrt{(x_t + D_{x})^2 + y_t^2} + \sqrt{(x_t - D_{x})^2 + y_t^2}}{2}, \quad c_t = D_{x}
\]

\[
(x + D_{x})^2 + y^2 = (x_t - D_{x})^2 + y_t^2
\]

\[
(x - D_{x})^2 + y^2 = (x_t + D_{x})^2 + y_t^2
\]

where the first expression is the equation for the ellipse, and the remaining two expressions in (2) are the equations for the circles. There exists two solutions to this system of equations; the first is the true target location \( x_t \), and the other is at \([−x_t, −y_t]^T\), which is behind the radar.

B. **Free-space model**

In practice, when considering multiple walls, such as those shown in Fig. 3, additional processing must be performed, which is not applicable in the single-wall scenario. In essence, the ToAs will need to be associated to the sources of multipath, namely walls-1, 2 and 3. Without correct associations, the ellipses and circles may not all intersect at the true target coordinates. Figure 3 represents an urban canyon type geometry in which the target is located at \( x_t \). Specular multipath is considered. The location and the extent, \( D_{i}, i=1,2 \), of the side and back walls are assumed known. The rest of the target and radar parameters are identical to those in Figs. 1 and 2. For clarity, the direct path is denoted as path-A, whereas the multipath from walls-1, 2, 3 is, respectively, denoted as paths-B, C, and D. Higher order multipaths,
which incorporate multiple reflections at the walls, are ignored as they suffer from severe fading and may be too weak to be observed [19]. Nonetheless, such higher order paths, if sufficiently pronounced, can be incorporated in a straightforward manner as demonstrated in [20].

The radar transmits a pulsed waveform $s(t)$, where '$t$' indexes the time within the pulse, and measures the received signal. The received signal is a superposition of the direct path and the multipath returns, given by

$$z(t) = \sum_{p \in \{A,B,C,D\}} \Gamma_p^2 s(t - \tau_p) + 2 \sum_{q \in \{B,C,D\}} \Gamma_q s(t - (\tau_p + \tau_q) / 2)$$

(3)

where $\tau_p$ and $\Gamma_p$ are, respectively, the ToA and the complex amplitude associated with reflection and transmission coefficients, for one-way propagation along path-$p$. The amplitude, $\Gamma_p$, can be readily derived, provided the material properties of the walls are known [4, 8]. Without loss of generality, we will assume that $\Gamma_p = 1, \forall p, p \in \{A,B,C,D\}$. From the geometry, it is clear that (3) includes a total of seven signal returns, which are assumed to be successfully detected. By virtue of propagation and reflections, the ToAs are naturally ordered in an ascending fashion, which may be represented in vector form as,

$${\boldsymbol{\tau}} = [\tau_1, \tau_2, \ldots, \tau_7]^T, \tau_1 = \tau_A, \tau_i < \tau_{i+1}, i = 1, 2, \ldots, 7$$

(4)

In (4), the first ToA always corresponds to the direct path. The rest of the ToAs are unascertained. For example, it is not known if $\tau_4$ is a first order or a second order multipath ToA, nor which particular wall is responsible for it. Therefore, two fundamental problems arise in multipath analyses; first, determining which ToAs are first order and which are second order, and how to pair the multipath returns that originate from the same wall; and second, determining which wall caused a particular pair of ToAs.

First, the problem of pairing the multipath returns is considered. Initially, we observe that the first ToA in $\boldsymbol{\tau}$, $\tau_1 = \tau_A$, is known to be the direct path and that the last ToA, $\tau_7$, must be a second order multipath. Next we note that if $\tau_p$, when $2 \leq p < 7$, is considered to be a first order multipath, only those $\tau_q$, where $p < q \leq 7$, could be the second order multipath ToA caused by the same wall. Using the
knowledge that ToA of a first order multipath is one half of the sum of the ToAs of the direct path and the matching second order multipath, e.g., for wall-1 in Fig. 3, \( \tau_{dB} = (\tau_d + \tau_g)/2 \), a cost function may be developed for the possible valid pairings of first and second order multipath. If a ToA \( \tau_p \), where \( 2 \leq p < 7 \), is a candidate for being a first order multipath, then for a second order multipath candidate, \( \tau_q \), where \( p < q \leq 7 \), from the same wall, we may write
\[
\left| (\tau_1 + \tau_q) / 2 - \tau_p \right| = \begin{cases} 0 & \text{if } \tau_q \text{ is the second order to } \tau_p \\ a & \text{where } a > 0, \text{ if } \tau_q \text{ is not the second order to } \tau_p \end{cases}
\]
assuming there is no error in the ToA measurements. By considering all valid pairs of \( \tau_p \) and \( \tau_q \) for \( p = 2, \ldots, 7 \), an ordered cost vector can be formed as
\[
\mathbf{\tau}_{\text{cost}} = \text{sort}\{[|\tau_d / 2 + \tau_3 / 2 - \tau_2 |, |\tau_d / 2 + \tau_4 / 2 - \tau_2 |, \ldots, |\tau_d / 2 + \tau_7 / 2 - \tau_2 |, \ldots, |\tau_d / 2 + \tau_6 / 2 - \tau_7 |]^T\}
\tag{5}
\]
where the operator \( \text{sort}\{\} \) arranges the elements in increasing order. The vector in (5) is comprised of 15 elements when three walls are considered. For three walls, the first three elements in (5) indicate the correct pairing of the first and second order multipath returns. From the first three elements of \( \mathbf{\tau}_{\text{cost}} \), we can extract “pair vectors”, \( \tau^{(k)}, k = 1, 2, 3 \), to store the associated multipath return ToAs. The structure of each \( \tau^{(k)} \) will be a two-element vector defined as \( \tau^{(k)} = [\tau_q^{(k)}, \tau_p^{(k)}]^T \), where \( \tau_q^{(k)} \) and \( \tau_p^{(k)} \) are the multipath ToAs from the \( k \)th element of \( \mathbf{\tau}_{\text{cost}} \). Before moving on, it is important to realize that the index \( k \) of \( \tau^{(k)} \) does not indicate which wall the ToAs are associated with.

Second, the problem of wall association is addressed. Ideally, when the ToAs are correctly associated, we have, for wall-1, the bistatic ellipse, corresponding to the first order multipath return, and monostatic circular contour centered at the virtual radar, corresponding to the second order multipath. These can be arranged in a vector format as,
\[
\begin{align*}
\mathbf{\beta}_1(\sigma_{dB}) &= \frac{4x^2}{\sigma_{dB}^2c^2} + \frac{4y^2}{(\sigma_{dB}^2c^2 - 4D_1^2)} - 1 = 0 \\
\mathbf{\beta}_2(\sigma_y) &= (x - D_3)^2 + y^2 - \sigma_y^2c^2 / 4 = 0 \\
\mathbf{\beta}_w &= [\beta_1(\sigma_{dB}), \beta_2(\sigma_y)]^T
\end{align*}
\tag{6}
\]
where $c$ is the speed of light in free space. Similarly, for wall-2 we have,

$$\beta_3(\tau_{AC}) = \frac{4(x + D_3)^2}{(\tau_{AC}^2 c^2 - 4D_3^2)} + \frac{4(y - D_2)^2}{\tau_{AC}^2 c^2} - 1 = 0$$

$$\beta_4(\tau_{D}) = (x + D_2)^2 + (y + 2D_2)^2 - \tau_{D}^2 c^2/4 = 0$$

$$\mathbf{b}_{w_2} = [\beta_3(\tau_{AC}), \beta_4(\tau_{D})]^T$$

Likewise, for wall-3,

$$\beta_5(\tau_{AD}) = \frac{4(x - D_1 + D_3)^2}{\tau_{AD}^2 c^2} + \frac{4y^2}{(\tau_{AD}^2 c^2 + 4D_1^2 - 4D_3^2)} - 1 = 0$$

$$\beta_6(\tau_{D}) = (x + 2D_1 - D_1)^2 + y^2 - \tau_{D}^2 c^2/4 = 0$$

$$\mathbf{b}_{w_3} = [\beta_5(\tau_{AD}), \beta_6(\tau_{D})]^T$$

Lastly, for the monostatic direct path, we have the circular constant range contour,

$$\beta_7(\tau_{A}) = (x + D_1)^2 + y^2 - \tau_{A}^2 c^2/4 = 0$$

The expressions defined in (6-9) can be concatenated into a single vector, $\mathbf{b}$, defined as

$$\mathbf{b} = [\mathbf{b}_{w_1}^T, \mathbf{b}_{w_2}^T, \mathbf{b}_{w_3}^T, \beta_7(\tau_{A})]^T$$

This vector is a function of the coordinates $x$ and $y$, combined into a vector $\mathbf{x}$, the three pairs of first and second order multipath ToAs, $\tau^{(k)}$, $k=1, 2, 3$, and the direct path ToA, $\tau_A$. In full, the vector-function would be written as $\mathbf{b}(\mathbf{x}, [\tau^{(1)^T}, \tau^{(2)^T}, \tau^{(3)^T}, \tau_A])$ but this full description has been omitted above for clarity.

Clearly, when the wall associations are correct and the ToAs estimates are perfect\(^1\), the circles and ellipses in (6-9) all intersect at one location and,

$$\|\mathbf{b}\|^2 = 0$$

However, for imperfect ToA measurements, we seek the vector $\mathbf{x}$ and the set of pairs of first and second order multipath ToAs that will minimize $\|\mathbf{b}\|^2$.

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\(^1\) Measured ToAs after matched filtering are quantized to belong to range gates, and adjacent range gates are separated by the system range resolution. Hence, in practice, the ellipses and circles do not all intersect at one common point but approximately intersect in a region.
The possible sets of pairs of first and second order multipath ToAs are limited. Consider the matrix, comprising all permutations of $\tau^{(k)^z}, k = 1, 2, 3$,

$$
\begin{bmatrix}
\tau^{(1)^z} & \tau^{(2)^z} & \tau^{(3)^z} \\
\tau^{(1)^z} & \tau^{(3)^z} & \tau^{(2)^z} \\
\tau^{(3)^z} & \tau^{(1)^z} & \tau^{(2)^z} \\
\end{bmatrix}
$$

for $k$ wall-1, wall-2, wall-3

$$
E_{perm} := \begin{bmatrix}
\tau^{(1)^z} & \tau^{(2)^z} & \tau^{(3)^z} \\
\tau^{(1)^z} & \tau^{(3)^z} & \tau^{(2)^z} \\
\tau^{(3)^z} & \tau^{(1)^z} & \tau^{(2)^z} \\
\end{bmatrix}
\in \mathbb{R}^{3x3}
$$

In (11), for example, the second row third column implies that $\tau^{(2)}$ was assigned to wall-3. Likewise, similar implications may be seen from (11). The elements of the row vectors $e_j$ now contain the possible sets of pairs of first and second order multipath ToAs. In the spirit of (10), the nonlinear least squares (NLS) error is minimum for the correct permutation in (11), i.e.

$$
\hat{x}_i = \min_{x=[x,y]^T,j} \left\{ \| \beta(x, e_j, \tau) \|^2 \right\}, j = 1, 2, \ldots, 6
$$

where the minimum is taken over all $j$. Equation (10) can be solved using numerical optimization. Note that the function to be minimized in (12) is over determined consisting of two unknowns and seven equations. Hence, target localization could still be achieved using the multipath returns alone even if the direct path was completely ignored.

### III. MULTIPLE TARGETS

Increasing the number of targets in the scene also increases the number of ToAs that need pairing and associating to walls. Fortunately, an approach similar to the single target case for identifying the first and second order multipath and associating the pairs to the walls can be applied. Consider $K$ targets. As in (4), the ToAs stored in the vector $\tau$ have a naturally increasing order, but now there are $7K$ elements, i.e.

$$
\tau = [\tau_1, \tau_2, \ldots, \tau_{7K}]^T, \tau_i < \tau_{i+1}, i = 1, 2, \ldots, 7K
$$

The first ToA, $\tau_1$, corresponds to the direct path return of the target closest in range to the radar. The rest of the ToAs are unascertained. Our approach is to first cluster the ToAs corresponding to the nearest target, and identify its first and second order multipath ToAs, and their association with the walls. This procedure
is then continued for the second nearest target, then the third, and so on until all K targets have been exhausted. This sequential process is depicted below.

1. **Initialize**: Let target count \( K_1 = 0 \) and set a temporary time delay vector, \( \tilde{\tau} = \tau \).

2. **Loop**: Repeat until \( K_1 = K \). For index \( k = 2, \ldots, 7(K - K_1) \) calculate and store the minimum cost of the set

\[
\{\tau_{\text{cost}}^k\} = \min\{(\tau_i / 2 + \tau_{k_i} / 2 - \tau_k), k_i \neq k, k_i \in \{2, 3, \ldots, 7(K - K_1)\}\}
\]

(14)

3. **Search**: Find and store the ToAs corresponding to the \( k_i \)-indices of the first three minima in the set \( \{\tau_{\text{cost}}^k\} \), as well as the ToAs associated with indices (\( k \)'s). The \( k_i \) indices represent the second order multipaths, and the \( k \)'s are locations of the associated first order multipaths in \( \tilde{\tau} \).

4. **Store**: Save the first ToA in \( \tilde{\tau} \), and the ToAs corresponding to the \( k_i \)-indices of the first three minima in the set \( \{\tau_{\text{cost}}^k\} \), and the corresponding ToAs indexed by the three \( k \)'s of step-3 in a vector denoted by \( \tilde{\tau}_{K_i} \).

5. **Associate and Localize**: Using the wall association procedure as described in Section II, compute the coordinates for the \( K_1 \)-th target.

6. **Modify**: Increment target counter as \( K_1 = K_1 + 1 \), since we have clustered the direct and multipath ToAs corresponding to the \( K_1 \)-th target in \( \tilde{\tau}_{K_i} \). Using a set difference operator, \( \{-\} \), remove the elements in \( \tilde{\tau}_{K_i} \) from \( \tilde{\tau} \) by,

\[
\tilde{\tau} = \tilde{\tau} \setminus \tilde{\tau}_{K_i}
\]

(15)

7. **Check**: Sort the elements in ascending order in the new vector, \( \tilde{\tau} \). Go back to step-2

It is noted that in step-2 of the algorithm and especially in (14), we considered only the minimum cost for every \( k \). The algorithm performs as desired when all the direct paths and all the multipaths corresponding to the \( K \) targets are resolved. However, if some of the direct paths and the multipaths are
unresolved, then the proposed clustering algorithm will fail. Unresolved returns may arise, for example, when a target is present on the constant range contour of another target. In this case, although there will be two identical ToAs belonging to separate radar-target geometries, a single peak will result in the cross-correlation output. Further, the number of multipath and direct path returns in the underlying localization problem is over determined. If we were able to know *a priori* which of the ToAs are unresolved, it would still be possible to cluster the ToAs for the different targets. We could then continue with localization provided that the number of unresolved ToAs is large enough not to render the localization problem as under-determined. In certain other plausible cases, although hard to imagine, the second ToA in (13) could in actuality correspond to the direct path of one of the targets. In this case, during the search in (14), if the first order or second order multipath ToAs are close to one half the sum of the direct path ToA and the second ToA in (13), then the algorithm fails to perform satisfactorily. Lastly, the localization may perform poorly when one of the target or multipath return is below the noise floor and fails detection.

In essence, the difficulty arising from unresolved returns is one of clustering and association, not of multipath based localization. Although not the focus of this paper, the use of a “smart” clustering and association algorithm that could estimate which ToAs were unresolved could overcome these shortfalls. Also, we note that, in the moving target scenario, such unresolvable returns are unlikely to persist, and target localization using the proposed approach is still possible.

IV. THROUGH-THE-WALL OPERATION

When operating through the wall, the free-space approach taken above is not suitable. The localization technique outlined in Section II depends on knowledge of the shape of the constant range contours. Even when the wall parameters are known, the constant range contours for a given ToA under TTW operation would not have closed form expressions. This prevents formulation of an NLS approach in a manner outlined above. Fortunately, as we will demonstrate in this section, an alternative NLS solution is possible that allows for incorporation of the effects of the wall. However, the TTW solution requires an iterative root finding operation to solve for angles of refraction associated with TTW propagation [12, 20] for every
candidate target location, which significantly increases the computational burden, and makes it sensitive to initial conditions. To circumvent these difficulties, a good quality initial guess should be used for the solution. A reasonable initial guess would be the location estimated by the free-space noncoherent approach under assumption of LOS propagation.

For TTW operation, the signal returns are non-LOS and the propagation inside the wall must be taken into account for accurate localization. Consider Fig. 4 which shows the model for a target in an enclosed structure. There is a front wall and walls-1, 2, 3. The front wall has known thickness and dielectric constant given by $d_i$ and $\varepsilon_i$, respectively. Similar to the free space case, we consider the direct path, referred to as path-A, and three additional paths, namely, paths-B, C, and D, which correspond to the multipaths. Note, however, that the ToAs are not identical to the free space propagation and now depend on the slowing of the wave within the front wall and the reflection and refraction that occurs at the wall boundaries [9, 12, 20].

For target localization, an NLS procedure can be initialized with the free-space solutions. Consider the vector $\tau_k, k = 1, 2, \ldots, K$, which consists of the ToAs corresponding to the direct path, and the first and second order multipath returns for the $k$-th target. Specifically,

$$\begin{align*}
\tau_k &= [\tau_{1k}, \tau_{2k}, \ldots, \tau_{7k}]^T, \\
\tau_{1k} &= \tau_{6k} + \tau_{2k} \\
\tau_{6k} &= \frac{\tau_{6k} + \tau_{4k}}{2}, \\
\tau_{7k} &= \frac{\tau_{4k} + \tau_{2k}}{2}
\end{align*}$$

(16)

The vector $\tau_k$ is obtained by the clustering algorithm outlined in Section III. In (16), the first time delay corresponds to the direct path of the $k$-th target, and is denoted likewise. The ToAs, $\tau_{2k}, \tau_{3k}$, and $\tau_{4k}$ correspond to the second order multipath, whereas the remaining ToAs in (16) are their first order multipaths. For multipath-wall association, we resort to (12) and apply NLS solutions for each target $k$. Initialization is performed by solving a similar free-space equation separately for each one of the six possible associations (permutations), i.e.,

$$x_{kj}^{FS} := \min_{x} \left\{ \beta(\mathbf{x}, e_{kj}, \tau_{kj}) \right\}, j = 1, \ldots, 6$$

(17)
where $\mathbf{e}_j, j = 1, \ldots, 6$ is the permutation vector for the $k$-th target. The difference between (11) and (17) is that in the former, minimization is performed over all $j$, whereas in the latter, minimization is performed for a particular $j$. In other words, (11) gives a single free-space solution, whereas (17) provides six free space solutions of the target position. The proposed algorithm is described below.

1. **Cluster:** Cluster the $k$-th TTW target’s direct path, first and second order multipath ToAs and store them in a vector $\tau_k$. The clustering is performed using the algorithm for multiple targets, and also identifies whether a particular ToA is either a first order or second order multipath.

2. **Compute:** Assuming a free space model, compute the various free space solutions, $\mathbf{x}_{ij}^{FS}$ as in (17) for all possible wall permutations. These free space solutions will be used to initialize the actual TTW NLS localization estimator.

3. **NLS:** For a particular $j$, consider the optimization, initialized with the free space solution, $\mathbf{x}_{ij}^{FS}$

$$\mathbf{x}_{ij}^{TTW} = \min_{\mathbf{x}, \tau} \left\| \mathbf{e}_j - \mathbf{r}_i \right\|^2, \text{ initialized with } \mathbf{x}_{ij}^{FS}, j = 1, 2, \ldots, 6$$

$$\mathbf{e}_j := [\tau_i^x, \tau_j^y]^T, \mathbf{x} := [x', y']^T$$

$$\tau := [\tau_1^x(\psi'_{iA}), \tau_2^x(\psi'_{iB}), \ldots, \tau_6^x(\psi'_{iD}), \tau_1^y(\psi'_{iA})/2 + \tau_2^y(\psi'_{iB})/2 + \tau_3^y(\psi'_{iC})/2 + \tau_6^y(\psi'_{iD})/2]^T$$

with $\mathbf{x}_{ij}^{FS}$, for which the vector $\tau'$ can be found by computing the angles, $\psi'_{ip}, p \in \{A, B, C, D\}$.

The angles can be computed by solving the equations numerically,

$$d_1 \tan(\psi'_{iA}) + (y' - d_1 \tan(\psi'_{iA})) - D_x + x' = 0$$

$$d_1 \tan(\psi'_{iB}) + (y' - d_1 \tan(\psi'_{iB})) - D_x = 0$$

$$(2D_2 - y') \tan(\psi'_{iC}) + d_1 (\tan(\psi'_{iC}) - \tan(\psi'_{iC})) - D_x + x' = 0$$

$$d_1 \tan(\psi'_{iD}) + (y' - d_1 \tan(\psi'_{iD})) - D_x + D_y = 0$$

$$\psi'_{ip} = \sin^{-1} \left( \frac{\sin(\psi'_{ip})}{\sqrt{E_i}} \right), p \in \{A, B, C, D\}$$

where $y'_{ip}$ and $y'_{iD}$ can be substituted by,

$$y'_{ip} = y' - x' \cot(\psi'_{ip})$$

$$y'_{iD} = y' - (D_i - x') \cot(\psi'_{iD})$$

(19)
Repeat the procedure \( \forall j \). The TTW \( k \)-th target location estimate, \( x_{jTTW}^{TW} \), is obtained as one which yields the minimum cost in (18) \( \forall j \).

4. **Check:** Stop if all the \( k \) targets have been localized, otherwise go to step-1 and repeat.

In step-4, without invoking unnecessary mathematical operators, and for meaningful interpretation of the NLS cost, it is assumed that the elements in \( e_{ij}' \) are ordered in an identical fashion as in \( \tau' \). In other words, in line with the current permutation, the first element in \( e_{ij}' \) is the direct path, the next three elements are the second order multipath ToAs, corresponding to paths-\( B, C, D \), while the remaining elements are their associated first order multipath ToAs. Furthermore, in (18), the elements of \( \tau' \) depend on the angles, \( \psi_{ip}', p \in \{A,B,C,D\} \). These angles are highly nonlinear functions of \( x' \). The angles are obtained by solving the equations in (19) using (20). We note that obtaining the angles includes an inherent optimization, albeit in a single variable for each path. In essence, (18) consists of an explicit NLS stage, and an implicit root finding stage to obtain the angles \([9, 12, 20]\), necessary to construct the ToAs in \( \tau' \).

For the correct wall association, it is readily seen that the minimum is obtained.

In the above algorithm, the free space solutions are essential to estimate the TTW target locations. It is noted that the above formulation involves wall association along with the NLS and the root finding to compute the precise through-wall angles. If one is confident that, for each of the \( k \) targets, the free space wall associations are correct when applied to the TTW scenario, then (17) may be circumvented, and the free space approach may be used directly to obtain the final free space solution. That is, one can use (11-12) to obtain \( x_{iFS} \). Our formulation for the TTW problem, however, is more general, and does not place complete confidence in the approach of using the free space wall association for the TTW ToAs.
V. SIMULATIONS AND EXPERIMENTAL RESULTS

A. Simulated Results

The target locations and dimensions of the walls etc. are all in meters unless noted otherwise. Consider a single target at $x_t = [-16.7, 8.6]^T$ in the scene identical to Fig. 3. The parameters $D_1 = 20$ and $D_2 = 15$ and the radar’s coordinates are given by $R = [-12, 0]^T$. Fig. 5 shows the various constant range ellipses and circles with respect to the walls-1,2,3 for the considered scene. In particular, the six subfigures, namely, Fig. 5(a-f), show the intersection of the various ellipses and circles for all possible permutations of the first and second order multipath ToAs as given in (11). The ToAs (hence ranges) used in Fig. 5(a-f) are exact and are derived from simple geometry. In these figures, the true target location is indicated by ‘□’, the actual radar and the virtual radar locations are all marked by ‘*’, and the origin is indicated by ‘O’. The walls are depicted as black dashed lines in the figures. By the convention adopted in the rest of this section, the virtual radars are in red, and the actual radar is in blue. Likewise, the elliptical bistatic constant range contours and the circular monostatic constant range contours centered at the virtual radars are plotted in red and blue (solid lines for wall-1, dashed lines for wall-2, and dashed-dotted lines for wall-3), respectively, whereas the monostatic circular constant range contour centered at the actual radar is shown by a solid black line. The six different permutations (or wall associations) can be inferred from the figure titles. For example, consider Fig. 5(a)’s title, which states that the respective ToAs of wall-1 have been assigned correctly to wall-1, whereas ToAs arising from reflections at wall-2 are assigned to wall-3 while wall-3’s ToAs are assigned to wall-2. The last permutation, shown in Fig. 5(f), corresponds to the correct wall associations.

The NLS costs in (12) for each of the permutations in Fig. 5 are provided in the first row of Table I. In Section II, for ease of exposition, there was an implicit assumption that the constant range contours, especially the bistatic ellipses, exist for all $3!$ permutations. However, this is not always the case. The ellipses associated with wall-2 in Fig. 5(a) and Fig. 5(e) have minor axes equal to zero. In actuality, they do not exist as the parameter proportional to the minor axis is purely imaginary. Hence the permutations from
these two combinations can be ignored while using the NLS of (12), yielding some computational savings. Removal of the nonexistent ellipses from the NLS has no effect on the values presented in Table I corresponding to Fig. 5. From this table, it is evident that the NLS of (12) would choose the last permutation to be the correct wall association yielding the estimated target coordinates, \( \hat{\mathbf{x}}_t = [-16.69, 8.6]^T \).

Figure 6 shows the results for the same scene as for Fig. 5, but with the target at position \( \mathbf{x}_t = [-4.7, 7.6]^T \). Only the result for the correct wall association is presented in Fig. 6. The corresponding NLS costs for all 6 permutations are provided in the second row of Table I, where the cost of the correct solution is observed to be several orders of magnitude less than the other possibilities.

Next, consider two targets at coordinates (1) \( \mathbf{x}_t^{(1)} = [-15.4, 8]^T \) and (2) \( \mathbf{x}_t^{(2)} = [-4.6, 5.6]^T \) in a scene similar to Fig. 3. The radar transmits a rectangular pulse with bandwidth equal to 1.2 GHz, and carrier frequency set at 2 GHz. The rest of the parameters are identical to the single target scenarios corresponding to Figs. 5 and 6. In Fig. 7(a), peaks of the range profile after matched filtering are shown. There are 20 peaks corresponding to the direct path and first and second order multipath returns for the two targets. It is noted that some of the peaks, although resolvable, are close to each other. Application of the processing from Section III leads to Fig. 7(b), which confirms that both targets have been localized correctly. The NLS estimates for the locations are given by \( \hat{\mathbf{x}}_t^{(1)} = [-15.39, 8.007]^T \) and \( \hat{\mathbf{x}}_t^{(2)} = [-4.60, 5.59]^T \). The errors in these solutions are in the second decimal place.

For the TTW scene shown in Fig. 4, consider two targets at locations \( \mathbf{x}_t^{(1)} = [-15.4, 7.6]^T \) and \( \mathbf{x}_t^{(2)} = [-4.6, 14.6]^T \). The parameters \( D_1 \) and \( D_2 \) are chosen as 20 and 25, respectively, whereas the standoff distance \( D_y = 4 \). The front wall has a thickness of 0.2m and dielectric constant of 7.6, and was assumed to be lossless. In order to increase realism, the associated (parallel polarization) reflection and transmission coefficients were incorporated in the simulation to mimic the losses due to reflections and transmission through and at the walls. These reflection and transmission coefficients have standard
derivations [21]. The rest of the parameters are identical to those of the free-space simulation in Fig. 7. The range profile peaks are shown in Fig. 8(a), and as in Fig. 7(a), some of the peaks are again close to each other. Fig. 8(b) was obtained by applying the free-space processing of Section II to the ToAs extracted from the range profile. In the figure, we can see that the free-space solution is close to the true solution but does not coincide with it. Specifically, the free-space solution for the first target is given by \([-15.75,7.37]\)^T and that for the second target is \([-4.30,14.58]\)^T; both have errors in the first decimal place. Note that these free-space solutions are obtained from the NLS using (17) for the correct wall association. Continuing the processing to include the TTW specific NLS as described in the algorithm consisting of (18, 19), we obtain the final TTW solutions as \([-15.39,7.60]\)^T and \([-4.59,14.61]\)^T. The errors are now in the second decimal place.

B. Experimental Results

As a final stage of the investigation, target localization was undertaken using experimental data. The scene, shown in Fig. 9, consists of a single target adjacent to a concrete wall. The origin was chosen to be at the wall, and the radar and target coordinates are given by (1.057m, 0m) and (0.75m, 4.54m), respectively. The radar was stepped frequency based and operated between 1-4GHz, with a frequency step-size of 3.749 MHz, giving rise to a maximum unambiguous range of 40m. The target was a metal sphere with radius 0.355m. The wall was made of solid concrete blocks, and while it is intended to be as straight and flat as possible, clearly has a bowed nature and several slight edges where the blocks meet.

The range profile obtained, after background subtraction and application of frequency domain Kaiser window with parameter set to unity and matched filtering, is shown in Fig. 10(a). The target ToA and the first and second order multipath ToAs are indicated by red arrows. There exists another peak between 5m-5.5m, which is comparable in strength to the second-order multipath and is attributed to a combination of the target-wall interactions and the creeping wave phenomenon [22]. The creeping wave return occurs \((2+\pi)r\) m after the target return, where \(r\) is the radius of the sphere. For the sphere used, the creeping wave return would appear at approximately 5.33m, which is in agreement with the results in Fig. 10(a). To
increase the accuracy of the estimated target and multipath return ToAs, spline interpolation was applied to the data, resulting in the range profile shown in Fig. 10(b). Since a single target was present with a single wall configuration, wall association and ToA clustering was unnecessary. Using the extracted target and multipath ToAs, the equations for the corresponding ellipse and circles were computed, and are depicted in Fig. 11. We observe that the ellipse and circles do not exactly intersect at a single point. The LS estimate of the target position is (0.81m, 4.39m), which has a normalized squared error of 0.02m² indicating accurate localization.

VI. CONCLUSIONS

In this paper, we have demonstrated that, by utilizing the multipath reflections, target localization may be achieved with a single sensor. We showed that by exploiting the multipath returns arising from a single wall to create an additional virtual sensor, the target could be considered part of a monostatic and a bistatic geometry. When the two geometries apply, an analytical solution for the target location was obtained. To localize a target correctly in the presence of multiple walls, as may arise in a TTW or in an urban canyon situation, the multipath ToAs must be correctly associated to their respective walls. An algorithm to perform such an association was devised. Furthermore, it was shown that in the case of multiple targets, localization is achievable provided the multipaths and direct paths are all resolved.

Extension of these noncoherent localization techniques to through-the-wall radar was demonstrated. A nonlinear least squares approach was developed which provides correct location estimates by allowing for the change in propagation velocity and the refraction that occur as the wave interacts with the front wall. Simulation and experimental results were presented which validated the proposed techniques.

ACKNOWLEDGMENT

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REFERENCES


Fig. 1 Simple multipath model from a single wall.

Fig. 2 Principle of localization.
Fig. 3 Free space model and scene.

Fig. 4 Through-the-wall (TTW) model for a single target.
Fig. 5. Wall permutations and associations. (Direct path: black solid line, multipaths w.r.t wall-1: solid blue and red lines, multipaths w.r.t. wall-2: dashed blue and red lines, multipaths w.r.t. wall-3: dashed-dotted blue and red lines).

![Diagram](image)

Fig. 6 Example of correct localization after wall associations for a single target. (Direct path: black solid line, multipaths w.r.t wall-1: solid blue and red lines, multipaths w.r.t. wall-2: dashed blue and red lines, multipaths w.r.t. wall-3: dashed-dotted blue and red lines).

![Diagram](image)

Table I NLS costs for a single target for associating the ToAs to the respective walls.

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<th>Permutation</th>
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<td>Fig. 6</td>
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Fig. 7(a) Correlation peaks (two target case)

Solutions after all associations and tagging

Fig. 7(b) Two target localization after clustering and wall associations. (Direct paths: black solid lines, multipaths w.r.t wall-1: solid blue and red lines, multipaths w.r.t. wall-2: dashed blue and red lines, multipaths w.r.t. wall-3: dashed-dotted blue and red lines).
Fig. 8(a) Correlation peaks (two target TTW case)

Solutions after all associations and tagging

Fig. 8(b) Example of a TTW two target case after using the free space localization. (Direct paths: black solid lines, multipaths w.r.t wall-1: solid blue and red lines, multipaths w.r.t. wall-2: dashed blue and red lines, multipaths w.r.t. wall-3: dashed-dotted blue and red lines).
Fig. 9 Experimental setup

Fig. 10 Range profiles of experiment, (a) after using kaiser window, (b) using spline interpolation after windowing.
Fig. 11 Noncoherent localization results for experiment, the true and estimated LS solutions are shown.